PHYC 581: High Energy Astrophysics
Fall 2018

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Comptonization:

If the evolution of the spectrum is primarily determined by Compton scattering, the process is referred to as "Comptonization". The plasma must be just sufficiently dense in this case so that other radiation processes such as Bremsstrahlung do not contribute extra photons into the system.

If the plasma is hot, then the exchange of energy per collision is greater if the matter is hotter than the radiation. Examples of sources in which such conditions are found include the hot gas in the vicinity of binary X-ray sources, the hot plasma in AGNs, the hot intergalactic gas in galaxy clusters, and the primordial plasma in the early universe before recombination.

Here, we try to build a simple picture of the Comptonization process. We will restrict our discussion to the regime where $kT \ll m_e c^2$ and $h\nu \ll m_e c^2$. First, recall the expression for the energy transfer to stationary electrons when $h\nu \ll m_e c^2$:

$$\frac{\Delta E}{E} = \frac{-h\nu}{m_e c^2}(1-\cos\theta) \qquad (\theta: \text{scattering angle})$$

The average energy loss of the photon then is:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{\int_0^{\pi} \frac{\Delta E}{E}\, d\sigma_T}{\sigma_T} \qquad d\sigma_T = \frac{\pi e^4}{m_e^2 c^4}(1+\cos^2\theta)\, d\cos\theta$$

This results in:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{-h\nu}{m_e c^2}$$

Next, the low-energy limit of the energy loss rate of the high-energy electrons in inverse Compton scattering off low-energy photons is:

$$P_{Comp} = \frac{4}{3}\sigma_T \left(\frac{v}{c}\right)^2 c\, U_{rad}$$

This energy is gained by the photon, with the average energy gain per collision given by:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\left(\frac{v}{c}\right)^2$$

If the electrons have a thermal distribution, we have:

$$\frac{1}{2} m_e \langle v^2 \rangle = \frac{3}{2} kT$$

As a result, the net energy change of the photon in one collision follows:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{-h\nu}{m_e c^2} + \frac{4kT}{m_e c^2}$$

If $4kT > h\nu$, energy is transferred to the photon. While, if $h\nu > 4kT$, energy is transferred to the electron. There will be no net energy transfer if $h\nu = 4kT$.

We are primarily concerned with the case where the electrons are hotter than the photons. The fractional increase in energy is $\frac{4kT}{m_e c^2}$ per collision in this case. If the region has electron density n_e and size L, the optical depth for Compton scattering (which becomes Thomson scattering in the limit under consideration) is:

$\tau = n_e \sigma_T L$

If $\tau \gg 1$, then photons undergo a random walk in escaping from the region. The net distance travelled by the photon then is $N^{\frac{1}{2}} (n_e \sigma_T)^{-1}$, where N is the number of scatterings, and $(n_e \sigma_T)^{-1}$ is the photon mean-free-path ℓ. The number of scatterings within the region is:

$N = (n_e \sigma_T L)^2 = \tau^2$

If $\tau \gg 1$, the number of scatterings is so large that the photon spectrum will be distorted by Compton scattering. The condition for significant distortion is $4y \gtrsim 1$, where:

$y = \frac{kT}{m_e c^2} \tau^2$

After N scatterings, the energy of the photon relative to its initial energy is:

$$\frac{E'}{E} = \left(1 + \frac{4kT}{m_e c^2}\right)^N$$

Since $4kT \ll m_e c^2$ is assumed, this can be written as:

$$\frac{E'}{E} \approx \exp(4y)$$

Once the photons have acquired an energy $E' = 4kT$ due to heating, there will be no net energy transfer further. The optical depth necessary for this to occur is found to be:

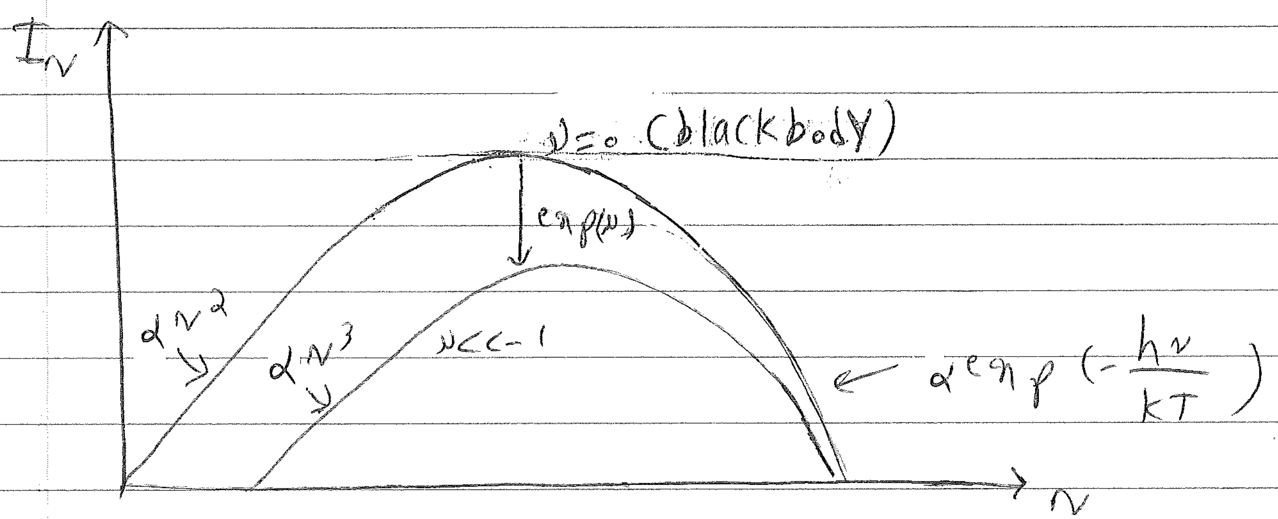
$$4kT = h\nu_0 \exp(4y) \Rightarrow \frac{4kT}{h\nu_0} = \exp\left[4\left(\frac{kT}{m_e c^2}\right)\tau^2\right] \Rightarrow$$

$$\tau = \left[\frac{m_e c^2}{4kT} \ln\left(\frac{4kT}{h\nu_0}\right)\right]^{\frac{1}{2}}$$

If the optical depth of the medium is greater than this, the photon distribution approaches its equilibrium value determined entirely by Compton scattering. The equilibrium spectrum is given by the Bose-Einstein distribution:

$$f = \frac{1}{\exp\left(\frac{h\nu}{kT} - \mu\right) - 1} \qquad \left(\begin{array}{l} f: \text{occupation number} \\ \mu: \text{chemical potential} \end{array}\right)$$

If $|\mu| \ll 1$, then the distribution will lead to the blackbody spectrum. In the opposite limit $|\mu| \gg 1$, the spectrum looks like a blackbody at high frequencies (i.e., in the Wien part), while it is modified in the Rayleigh-Jeans part at low frequencies. An illustration of the spectra with $\mu = 0$ and $\mu \ll -1$ is given in the figure below;



Next, we will discuss how the evolution of the spectrum toward equilibrium can be described.